

FST 10.3 Notes

Topic: The Binomial Theorem

GOAL

Explain the connections between combinations, Pascal's Triangle, and binomial coefficients and use the binomial theorem to solve certain counting problems in preparation for the next lesson.

SPUR Objectives

A Expand binomials using the Binomial Theorem.

D Interpret and describe properties of binomial coefficients combinatorially and algebraically.

K Represent combinations and binomial coefficients by Pascal's Triangle.

Vocabulary

expansion of $(x+y)^n$

binomial coefficients

binomial coefficients The coefficients in the expansion of $(x+y)^n$; the combinations ${}_n C_r$.

Mental Math

Multiply.

a. $(4+8)(4+8)$

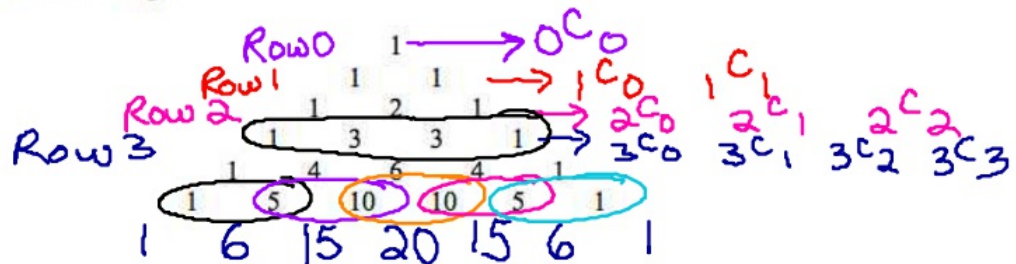
b. $(x+y)(x+y)$

c. $(40+1)(40+1)$

$(12)(12) = 144$
 $x^2 + 2xy + y^2$
 $(41)(41) = 1681$

	x	y
x	x^2	xy
y	xy	y^2

Review Pascal Triangle



a) How do we get to the next row?

Add 2 numbers above

b) How are the values in the triangle related to ${}_n C_r$?

$n = \text{row \#}$

$r =$ begins with 0, counts up by 1 and ends when it matches the row #

Review Expanding of binomials

1) $(x+y)^0 = 1$

2) $(x+y)^1 = x+y$

3) $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

4) $(x+y)^3 = (x+y)(x+y)(x+y)$

	x^2	$2xy$	y^2
x	x^3	$2x^2y$	xy^2
y	x^2y	$2xy^2$	y^3

$(x^2 + 2xy + y^2)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3$

If we try to expand $(x+y)^4$ it gets more cumbersome and difficult algebraically. The binomial theorem allows us to expand a power of a binomial.

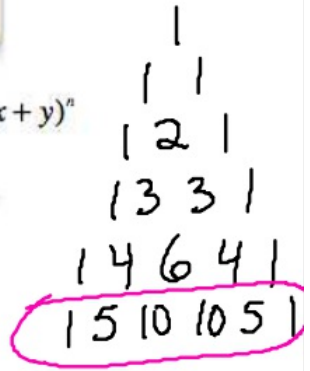
$(x+y)^3 = {}^3C_0x^3y^0 + {}^3C_1x^2y^1 + {}^3C_2x^1y^2 + {}^3C_3x^0y^3$

Binomial Theorem
 For any nonnegative integer n ,

$$(x+y)^n = {}_nC_0x^ny^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \dots + {}_nC_kx^{n-k}y^k + \dots + {}_nC_nx^0y^n$$

$$= \sum_{k=0}^n {}_nC_kx^{n-k}y^k.$$

**Each row of Pascal's triangle gives the coefficients for the expansion of $(x+y)^n$ for positive integers n .



For example, $(x+y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3x^1y^3 + {}_4C_4x^0y^4$
 $= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$

Using the Binomial Theorem expand the following expressions.

Expand $(x+y)^5 = {}^5C_0x^5y^0 + {}^5C_1x^4y^1 + {}^5C_2x^3y^2 + {}^5C_3x^2y^3 + {}^5C_4x^1y^4 + {}^5C_5x^0y^5$

$1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5$

Example 3

b) Find all terms in $(H + T)^5$ in which the power of H is at least two. *2 or more*

Power of H	Power of T	Product Type	Coefficients
2	3	$H^2 T^3$	$5^C_3 = 10$
3	2	$H^3 T^2$	$5^C_2 = 10$
4	1	$H^4 T^1$	$5^C_1 = 5$
5	0	$H^5 T^0$	$5^C_0 = 1$

$$10H^2T^3, 10H^3T^2, 5H^4T^1, 1H^5$$